Question II:

A. Use logarithmic differentiation to find $y'$ if $y = (\sqrt{2x+1}\, sinhx)^x$.

$$y = (\sqrt{2x+1}\ \sinh x)^x$$

$$\ln y = \ln(\sqrt{2x+1}\ \sinh x)^x$$

$$\ln y = x\ln\left[(2x+1)^{\frac{1}{2}}\sinh x\right]$$

$$\ln y = x\ln(2x+1)^{\frac{1}{2}} + x\ln(\sinh x)$$

$$D_x \ln y = D_x\left[\tfrac{1}{2}x\ln(2x+1)\right] + D_x\left[x\ln(\sinh x)\right]$$

$$(y)\ \frac{1}{y}y' = \left[\frac{1}{2}x\cdot\frac{1}{2x+1}(2) + \frac{1}{2}\ln(2x+1)\right] + \left[x\cdot\frac{1}{\sinh x}\cdot\cosh x + 1\cdot\ln(\sinh x)\right](y)$$

$$y' = \left[\left[\frac{2x}{2(2x+1)} + \frac{1}{2}\ln(2x+1)\right] + \left[\frac{x\cosh x}{\sinh x} + \ln(\sinh x)\right]\right]\left[(\sqrt{2x+1}\ \sinh x)^x\right]$$

2.5/2.5

B. Use the following reduction formula to evaluate $\int cos^5x\,dx$.

$$\int cos^n x\,dx = \frac{1}{n}cos^{n-1}x\,sinx + \frac{n-1}{n}\int cos^{n-2}x\,dx$$

1.5/1.5

$$\int \cos^5 x\,dx = \frac{1}{5}\cos^4 x\sin x + \frac{4}{5}\int \cos^3 x\,dx$$

$$= \frac{1}{5}\cos^4 x\sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x\sin x + \frac{2}{3}\int \cos x\,dx\right.$$

$$= \frac{1}{5}\cos^4 x\sin x + \frac{4}{15}\cos^2 x\sin x + \frac{8}{15}\sin x + C$$

Question I:

Choose the correct answer, then fill in the table below:

| Question Number | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Answer | b | a | c | a | b |

(1) $\frac{\log_2 x}{\log_3 x}$ equals  $\frac{1}{\ln 3}\cdot\frac{1}{x}$  $\frac{1}{x\ln 2} \div \frac{1}{x\ln 3}$  $\frac{x\ln 3}{x\ln 2}$

(a) $\frac{\ln 2}{\ln 3}$  (b) $\frac{\ln 3}{\ln 2}$  (c) $\frac{\log_2 3}{\log_3 2}$  (d) None of the previous

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(2) $\ln(\cosh^2 x - \sinh^2 x)$ equals  $\ln(1)$

(a) 0  (b) $e$  (c) 1  (d) None of the previous

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(3) $\cosh(\ln 2)$ equals  $\frac{e^{\ln 2} + e^{-\ln 2}}{2}$  $\frac{2+\frac{1}{2}}{2} = (2+\frac{1}{2})\cdot\frac{1}{2}$  $\frac{4\times 1}{4\times 1}+\frac{1}{4}$  $\frac{4}{4}+\frac{1}{4}$

(a) 2  (b) 0  (c) $\frac{5}{4}$  (d) None of the previous

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(4) $\int \frac{\ln e^x}{x^2}\,dx$ equals  $\int \frac{x}{x^2}\,dx$  $\int \frac{1}{x}\,dx = \ln x + C$

(a) $\ln|x| + c$  (b) $-\frac{x}{2} + c$  (c) $\frac{-1}{xe^x} + c$  (d) None of the previous

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(5) If $y = e^x + x^e$, then $\frac{dy}{dx}$ equals

(a) $e^x + x^e \ln x$  (b) $e^x + ex^{e-1}$  (c) $xe^{x-1} + ex^{e-1}$  (d) None of the previous

$y' = e^x + ex^{e-1}$

$y = e^x + Dx\,x^e$

Question III:

Evaluate the following integrals

(1) $\int x^4 lnx \; dx$

$u = \ln x \qquad dv = x^4\, dx$

$du = \frac{1}{x}\, dx \qquad v = \frac{x^5}{5}$

$= uv - \int v\, du$

$= \ln x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5}\, dx$

$= \ln x \cdot \frac{x^5}{5} - \frac{1}{5} \int \frac{x^5}{x}\, dx$

$= \ln x \cdot \frac{x^5}{5} - \frac{1}{5} \int x^4\, dx$

$= \ln x \cdot \frac{x^5}{5} - \frac{1}{5} \cdot \frac{x^5}{5} + c$

$= \frac{x^5}{5} \ln x - \frac{1}{25} x^5 + c$

(2) $\int 7^x \sec 7^x\, dx$

$u = 7^x$

$\frac{1}{\ln 7} du = \frac{\cancel{\ln 7} \cdot 7^x\, dx}{\cancel{\ln 7}}$

$\therefore I = \frac{1}{\ln 7} \int \sec u\, du$

$= \frac{1}{\ln 7} \cdot \ln|\sec u + \tan u| + c$

$= \frac{1}{\ln 7} \ln|\sec 7^x + \tan 7^x| + c$

m odd

(3) $\int \frac{1}{\csc^3 x} \cos^2 x \, dx \; = \int \sin^3 x \cos^2 x \; dx$

$= \int \sin^2 x \cos^2 x \; \sin x \; dx$

$u = \cos x$

$\frac{du}{-1} = \frac{-\sin x \, dx}{-1}$

$= \int (1-\cos^2 x) \cos^2 x \; \sin x \; dx$

$\therefore I = -\int (1-u^2) u^2 \; du$

$= -\int u^2 - u^4 \; du$

$\frac{2}{2}$

$= -\int u^2 du + \int u^4 \; du$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C \qquad = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

(4) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx \qquad \int \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}} \; dx$

Let $u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}} \; dx$

$= \int u \; du$

$= \frac{u^2}{2} + C$

$= \frac{(\sin^{-1} x)^2}{2} + C$

$\frac{1.5}{1.5}$

$D_x \tan^{-1} x = \frac{1}{a^2+u^2}$

$D_x \tanh^{-1} x = \frac{1}{a^2-u^2}$

(5) $\int \frac{e^x}{16-e^{2x}} dx \qquad \int \frac{e^x}{4^2-(e^x)^2}$

$= \int \frac{1}{4^2-u^2} du \qquad u = e^x \quad du = e^x dx$

$= \frac{1}{4} \tanh^{-1} \frac{u}{4} + C$

$= \frac{1}{4} \tanh^{-1} \frac{e^x}{4} + C$

1.5

(6) $\int_0^1 \frac{1}{\cosh^2(3x)} dx = \int \operatorname{sech}^2(3x)\, dx \qquad$ Let $u = 3x$

$\frac{1}{3} du = \frac{\cancel{3}}{\cancel{3}} dx$

$\therefore I = \frac{1}{3} \int \operatorname{sech}^2 u \, du$

$= \frac{1}{3} \tanh u + C$

2

$= \frac{1}{3} \tanh 3x + C$

$\int_0^1 \frac{1}{\cosh^2(3x)} dx$

$= \frac{1}{3} \tanh 3x \Big]_0^1$

$= \left[ \frac{1}{3} \tanh 3(1) - \frac{1}{3} \tanh 3(0) \right]$

$= \left[ \frac{1}{3} \tanh 3 - \frac{1}{3} \tanh 0 \right]$

Good Luck☺