King Saud University
Science and Medical Studies Section for girls
College of Science
Department of Mathematics

Math 106
2ND Semester 1437-36
1st Midterm
90 Minutes

Student's Name:	Student ID.:
Group No.:	Teacher's Name:

Question No.	I	II	III	IV	Total
Mark					

QUESTION I

A. Choose the correct answer :

1. An antiderivative of $f(x) = \sin x \cos x$ is

i. $-\cos x \sin x$	ii. $\frac{1}{2}\sin^2 x$ ✓	iii. $\cos x \sin x$	iv. None of the answers

2. If $\int_1^4 \sqrt{x}dx = \frac{14}{3}$ then $\int_1^4 \sqrt{x}dx + \int_4^1 \sqrt{x}dx$ is equal to:

i. 0 ✓	ii. $\frac{-7}{3}$	iii. $\frac{28}{3}$	iv. None of the answers

3. $\lim_{n\to\infty}\left(\frac{1}{n^4}\right) \sum_{k=1}^{n}(4k^3)$ is equal to:

i. 1 ✓	ii. $\frac{1}{4}$	iii. 0	iv. None of the answers

4. $\frac{d}{dx}\int_0^x \sqrt{t^2+1}\, dt$ is equal to:

i. $2(t^2+1)/3$	ii. 0	iii. $\sqrt{t^2+1}$	iv. $\sqrt{x^2+1}$ ✓

1) $\int f(x)dx = \int \sin x \cos x \, dx$

let $u = \sin x$

$du = \cos x \, dx$

$I = \int u \, du = \frac{u^2}{2} + c$

$= \frac{\sin^2 x}{2} + c$

2) if $\int_1^4 \sqrt{x}\, dx = \frac{14}{3}$

then $\int_1^4 \sqrt{x}\, dx + \int_4^1 \sqrt{x}\, dx$

$= \int_1^4 \sqrt{x}\, dx - \int_1^4 \sqrt{x}\, dx$

$= \frac{14}{3} - \frac{14}{3} = 0$

3) $\lim_{n\to\infty} \frac{1}{n^4} \sum_{k=1}^{n} 4k^3$

$= \lim_{n\to\infty} \frac{\cancel{4}}{\cancel{n^4}\, n^2} \frac{\cancel{n^2}(n+1)^2}{\cancel{4}}$

$= \lim_{n\to\infty} \frac{n^2+2n+1}{n^2} = 1$

4) $\frac{d}{dx}\int_0^x \sqrt{t^2+1}\, dt = \sqrt{x^2+1}$

B. **Without evaluating the integrals, show that** $\int_2^3 \frac{1}{x^2+3}dx \le \int_2^3 \frac{1}{x^2}dx$

Since $x \in [2,3]$

$x^2 = x^2$

$x^2+3 \geq x^2 \quad \forall x \in [2,3]$

$\frac{1}{x^2+3} \le \frac{1}{x^2} \quad \forall x \in [2,3]$

by theorem $\int_2^3 \frac{1}{x^2+3}\,dx \le \int_2^3 \frac{1}{x^2}\,dx.$

QUESTION II

A. **Compute the area of the region between the curve** $y = 3x+1$ **and the x-axis on** $[0,1]$ **using the limit of Riemann sum.**

$f(x) = 3x+1 \qquad a=0 \qquad b=1$

1) $\Delta x = \frac{1-0}{n} = \frac{1}{n}$

2) $x_k = a + k\Delta x = 0 + k \cdot \frac{1}{n} = \frac{k}{n}$

3) $f(x_k) = \frac{3k}{n} + 1$

4) $\text{Area} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3k}{n}+1\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3k}{n^2} + \frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{3}{n^2} \sum_{k=1}^{n} k + \frac{1}{n} \sum_{k=1}^{n} 1$

$= \lim_{n\to\infty} \frac{3}{n^2} \frac{n(n+1)}{2} + \frac{1}{n} \cdot n$

$= \lim_{n\to\infty} \frac{3}{2} \cdot \frac{n+1}{n} + 1 = \frac{3}{2} \cdot 1 + 1 = \frac{5}{2}$

$\int_0^1 (3x+1)\,dx$

$= \frac{3x^2}{2} + x \Big]_0^1$

$= \left(\frac{3}{2}+1\right) - (0)$

$= \frac{5}{2}.$

B. Find the value of c that satisfies the Integral Mean Value Theorem for

$f(x)=\sqrt[3]{x+1}$ **on** $[-2,0]$

$\because f(x)=\sqrt[3]{x+1}$ is continuous on $[-2,0]$

their is $c\in(-2,0)$ such that

$f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$

$\sqrt[3]{c+1}=\frac{1}{0-(-2)}\int_{-2}^{0}\sqrt[3]{x+1}\,dx$

$\sqrt[3]{c+1}=\frac{1}{2}\cdot 0$

$\sqrt[3]{c+1}=0$

$c+1=0$

$c=-1\in(-2,0)$

$\therefore c=-1$

$\int_{-2}^{0}\sqrt[3]{x+1}\,dx$

$=\int_{-2}^{0}(x+1)^{1/3}dx$

$=\frac{3}{4}(x+1)^{4/3}\Big]_{-2}^{0}$

$=\frac{4}{3}\left[(1)^{4/3}-(-1)^{4/3}\right]$

$=\frac{4}{3}\cdot 0=0$

QUESTION III:

A. If $F(x)=\int_0^x \sin^2 t\,dt$ **. Show that** $F'(x)+\cos^2 x=1$.

Solution: $F'(x)+\cos^2 x=1$??

$F(x)=\int_0^x \sin^2 t\,dt$

$F'(x)=\sin^2 x$.

$\therefore F'(x)+\cos^2 x=\sin^2 x+\cos^2 x=1$

B. Find the area of the region bounded by the x-axis, the function $Y=X^2$, $X=1$, $X=2$.

$$\text{Area} = \int_1^2 x^2\,dx$$

$$= \left.\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}.$$

<u>QUESTION IV :</u> Evaluate the following integrals

1) $\displaystyle\int_4^9 \frac{1}{x\sqrt{x}}dx$

$$I = \int_4^9 \frac{1}{x \cdot x^{1/2}}\,dx$$

$$= \int_4^9 \frac{1}{x^{3/2}}\,dx$$

$$= \int_4^9 x^{-\frac{3}{2}}\,dx$$

$$= \left. -2x^{-\frac{1}{2}}\right]_4^9$$

$$= \left.\frac{-2}{\sqrt{x}}\right]_4^9$$

$$= \left(\frac{-2}{\sqrt{9}}\right) - \left(\frac{-2}{\sqrt{4}}\right)$$

$$= \frac{-2}{3} + \frac{2}{2} = \frac{-2}{3} + 1 = \frac{1}{3}$$

2) $\displaystyle\int \frac{2x^2 - x}{x}dx$

$I = \int \left(\frac{2x^2}{x} - \frac{x}{x}\right) dx$

$= \int (2x - 1) dx$

$= \frac{2x^2}{2} - x + C$

$= x^2 - x + C$

3) $\displaystyle\int \frac{\sin x}{\sqrt{1-\cos x}}dx$

let $u = 1 - \cos x$

$du = -(-\sin x)dx$

$du = \sin x \; dx$

$1 - \frac{-1}{2} = \frac{1}{2}$

$I = \int \frac{1}{\sqrt{u}} du$

$= \int u^{-\frac{1}{2}} du$

$= 2u^{1/2} + C$

$= 2\sqrt{u} + C$

$= 2\sqrt{1-\cos x} + C$