106Midterm1grading scheme(Sem1-40/41)

<u>Question1</u> a) $\sum_{11}^{30} k^2 = \sum_{1}^{30} k^2 - \sum_{1}^{10} k^2 = 20c \quad (\mathbf{1})$

$9455 - 385 = 20c \;\; so\; c = \dfrac{907}{2} = \;\; 453.5 \;\; (\mathbf{1})$

b) $S_6 = \frac{2\pi-0}{3.8}(1 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2(-1)^4 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0$

$$+ 4\left(\frac{\sqrt{2}}{2}\right)^4 \;\; + 1) \;\; (\mathbf{2})$$

$$= \frac{2\pi}{3} \approx 2.09439 \;\; (\mathbf{1})$$

c) $P = \{0, \frac{1}{n}, \frac{2}{n}, \ldots\ldots 1\}\; x_k = \frac{k}{n} \; and \; \Delta x_k = \frac{1}{n}$, take $u_k = x_k \;\; 1 \le k \le n \;\; (\mathbf{1})$.

$$R_P = \sum_{1}^{n} (\frac{k}{n})^3 \frac{1}{n} = \frac{1}{n^4} (\frac{n(n+1)}{2})^2 \to \frac{1}{4} \;\; as\; n \to \;\; so \;\; \int_0^1 x^3 dx = \frac{1}{4} \;\; (\mathbf{2})$$

<u>Question2</u> a) $\int \frac{5^{tanx}}{(cosx)^2} dx = \int 5^u du \;\; u = tanx \;\; (\mathbf{2})$

$$= \frac{1}{ln5} 5^{tanx} + C \; (\mathbf{1})$$

$b)\; y' = 2ln2sinxcosx2^{(sinx)^2} + \pi x^{\pi-1}\pi^x + ln\pi x^\pi \pi^x$

$$(\mathbf{1}) + \frac{\mathbf{1}}{\mathbf{2}} + (\frac{\mathbf{1}}{\mathbf{2}})$$

$c) \int \dfrac{dx}{\sqrt{x}(2+x)} = 2\int \dfrac{du}{2+u^2} \;\;\; u = \sqrt{x} \;\; (\mathbf{2})$

$$= \sqrt{2}\tan^{-1}(\sqrt{x/2}) + C \; (\mathbf{1})$$

Question3 a) $\int \frac{lnx+1}{\sqrt{16(xlnx)^2-9}}dx = \frac{1}{4}\int \frac{du}{\sqrt{u^2-9}}$ $u = 4xlnx$ $(\mathbf{2})$

$= \frac{1}{4}\cosh^{-1}(\frac{4xlnx}{3}) + C$ $(\mathbf{1})$

b) $\int \frac{dx}{x\sqrt{x^5-4}} = \frac{2}{5}\int \frac{du}{u\sqrt{u^2-4}}$ $u = x^{5/2}$ $(\mathbf{2})$

$= \frac{2}{10}\sec^{-1}(\frac{x^{5/2}}{2}) + C$ $(\mathbf{1})$

c) $\int \frac{2e^{-3x}}{1-e^{-6x}}dx = \frac{-2}{3}\int \frac{du}{1-u^2}$ $u = e^{-3x}$ $(\mathbf{2})$

$= \frac{-2}{3}\tanh^{-1}(e^{-3x}) + C$ $(\mathbf{1})$

Note:the last answer implicitly assumes $x > 0$. Also accept the answer

$\frac{-2}{3}\coth^{-1}(e^{-3x}) + C$