106Midterm2 Solutions (Sem2-36/37)

Exercise1

a) $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{lnx}\right)=\lim_{x\to 1}\frac{lnx-x+1}{(x-1)lnx}=\lim_{x\to 1}\frac{\frac{1}{x}-1}{lnx+\frac{x-1}{x}}$ $\mathbf{(1.5)}$

$=\lim_{x\to 1}\frac{1-x}{xlnx+x-1}=\lim_{x\to 1}\frac{-1}{lnx+2}=-\frac{1}{2}$ $\mathbf{(1.5)}$

b) $\int e^x sin4xdx = e^x sin4x - 4\int e^x cos4xdx$ $\mathbf{(1)}$

$\int e^x cos4xdx = e^x cos4x + 4\int e^x sin4xdx$ $\mathbf{(1)}$

$\int e^x sin4xdx = \frac{1}{17}(e^x sin4x - 4e^x cos4x) + C$ $\mathbf{(1)}$

c) $\int sec^6 xdx = \int (1+tan^2x)^2 sec^2xdx$ $\quad u = tanx\ , du = sec^2xdx$ $\mathbf{(1)}$

$= \int (1+u^2)^2 du = \int u^4 + 2u^2 + 1du$ $\mathbf{(1)}$

$= \frac{1}{5}u^5 + \frac{2}{3}u^3 + u + C = \frac{1}{5}tan^5x + \frac{2}{3}tan^3x + tanx + C$ $\mathbf{(1)}$

Exercise2

a) $\int \frac{dx}{\sqrt{x^2+6x}} = \int \frac{dx}{\sqrt{(x+3)^2-9}} = \int \frac{du}{\sqrt{u^2-3^2}} = \cosh^{-1}\left(\frac{x+3}{3}\right) + C$ $\mathbf{(1)+(1)}$

b) $x = secu\ \ dx = secu.tanudu$ gives

$$\int \frac{dx}{x^3\sqrt{x^2-1}} = \int \frac{secu.tanudu}{sec^3u.tanu} \quad \mathbf{(1)}$$

$= \int cos^2udu \quad = \frac{1}{2}\int (1+cos2u)du$ $\mathbf{(1/2)}$

$$= \frac{1}{2}(u + sinu.cosu) + C \quad (\mathbf{1/2})$$

$$= \frac{1}{2}\left(\sec^{-1} x + \frac{\sqrt{x^2-1}}{x^2}\right) + C \ (\mathbf{1})$$

c) $\frac{2x^2-2x+2}{(1-x)(1+x^2)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1} = \frac{(a+b)x^2 + (c-b)x + a - c}{(x-1)(x^2+1)}$

$a = -1, b = -1 \text{ and } c = 1 \quad (\mathbf{1.5})$. We get

$$\int \frac{2x^2 - 2x + 2}{(1-x)(1+x^2)} dx = -\ln|x-1| - \frac{1}{2}\ln(x^2+1) + \tan^{-1} x + C \quad (\mathbf{1.5})$$

Exercise3

a)$u = \tan\left(\frac{x}{2}\right)$, $cosx = \frac{1-u^2}{1+u^2}$ $and\ dx = \frac{2du}{1+u^2}$ $(\mathbf{1})$

$$\int \frac{dx}{5 + 3cosx} = \int \frac{du}{4 + u^2} \quad (\mathbf{1})$$

$$= \frac{1}{2}\tan^{-1}(\frac{u}{2}) + C = \frac{1}{2}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right) + C \ (\mathbf{1})$$

b) $\int_0^c xe^{-2x^2} dx = -(\frac{1}{4}) \int_0^c -4xe^{-2x^2} dx = -\left(\frac{1}{4}\right)[e^{-2x^2}]_0^c$

$$= \frac{1}{4}\left(1 - e^{-2c^2}\right) \ (\mathbf{1})$$

$$\int_0^{\infty} xe^{-2x^2} dx = \lim_{c\to\infty} \int_0^c xe^{-2x^2} dx = \frac{1}{4} \ (\mathbf{1})$$

The integral converges.

c) Intersection points:

$y^2 - 1 = y + 1 \ \ so\ y = -1 or\ y = 2 \qquad (\mathbf{1})$

$$A = \int_{-1}^{2} (y - y^2 + 2)dy = \left[\frac{1}{2}y^2 - \frac{1}{3}y^3 + 2y\right]_{-1}^{2} = \frac{9}{2} \quad (\mathbf{1})$$

x = y + 1 (i)

$x = y^2 - 1$